Nonlinear Inversion

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LONG-TERM GOALS

The long term goals of this research are to develop practical and efficient algorithms for application to the nonlinear inversion problems encountered in ocean acoustics. Such algorithms would be used for estimating or accounting for the effects of the environment on acoustic propagation, detection and tracking in shallow water.

OBJECTIVES

The specific objectives of this research are to examine the sources of the nonlinearities, for acoustic inversion of shallow water environmental properties, and to assess their mitigation by means of a singular value decomposition of the model Jacobian matrix.

APPROACH

Many inverse problems of interest in ocean acoustics are intrinsically nonlinear, e.g. inverting measured pressure data for bottom and scattering properties. The solution to the nonlinear inversion problem is usually approached in one of two ways. The first way is to assume a starting model, which one hopes is near to the true model, then recursively solve a linearized version of the inverse problem for corrections to the starting model and model covariance. The advantage of this approach is that the numerical implementation of the solution algorithm is relatively straightforward, and in a linear problem the statistical properties are well defined and will remain gaussian if they start out gaussian. However linearization of a nonlinear system can produce biased estimates for two reasons: 1. Linearization of the system and/or measurement equations may not be a good approximation, and 2. Nonlinear systems do not maintain gaussian statistics as they evolve even if they are initially gaussian. Another problem with linearizing a nonlinear system is that with a poor starting guess the solution algorithm may never converge to the true answer. If the starting model represents a point near a local minimum of the solution space, the final solution will be trapped in that local minimum, and never converge to the true answer. This can be circumvented by using Monte Carlo techniques to randomly sample the solution space for starting models.

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The other class of solution methods attack the nonlinear problem directly by using simulated annealing or genetic algorithms. The disadvantage of these directly nonlinear methods, is that there is no way to conveniently propagate the statistical properties of the solution through to the final result. One solution to this problem is to find the global minimum in the solution space, if one exists, then linearize about the solution representing the global minimum and do a statistical analysis about that solution. This was done by Potty et al.(2000), who employed a genetic algorithm followed by linear analysis about the solution determined by the genetic algorithm.

In any case, a careful analysis of the linear problem leads to insight into the larger structure of the problem. One approach is to employ singular value decomposition (SVD) to make a first estimate of how much information is in and can be extracted from the data.

WORK COMPLETED

This project terminated in early calendar year 2010, and focused on the estimation of seafloor geoacoustic properties as a (possibly piece-wise) function of depth from pressure timeseries on an array of receivers in the water. The idea is to allow the data to fully specify the structure of the bottom, including smooth variation between discontinuities due to layering, rather than arbitrarily impose the assumption of a few homogeneous layers. An iterative linearization approach to the inversion of travel times is commonly used for geophysical and acoustic problems, but examples in the literature (Menke, 1989; Potty, 2000) show that for some seismoacoustic problems, fullwave inversion of the pressure timeseries itself can succeed with a higher-resolution result if the initial estimate is close enough to the solution point. In this work we have been exploring the use of linearized continuous fullwave inversion for the geoacoustic problem, examining for this application ways to learn the locations of discontinuities to implement in the regularization. We also have been studying whether the initial estimate can be realistically “distant” from the solution, and if not, whether an initial travel time inversion or signal-envelope inversion can provide a close enough initial estimate.

RESULTS

Figure 1 shows an analysis of the problem via singular value decomposition (SVD) of a Jacobian matrix of derivatives, those of the acoustic pressure timeseries on receivers of an ocean surface HLA array with respect to a P-wave velocity profile. The source (pulsed 100Hz signal with a 50Hz bandwidth) is located at the surface as well. For the sake of analysis, receivers in the HLA are every 50m from 50m to 2km in range from the source, and the 200m deep ocean is isovelocity at 1500m/s. The seafloor being estimated is parameterized down to 300m. The scenario is that we begin from an initial estimate of the bottom profile, in this case the constant gradient one shown in the dashed line in the lower left plot of Figure 1. The data in this synthetic problem was produced from the more complicated profile with discontinuities, the solid line in that same plot.

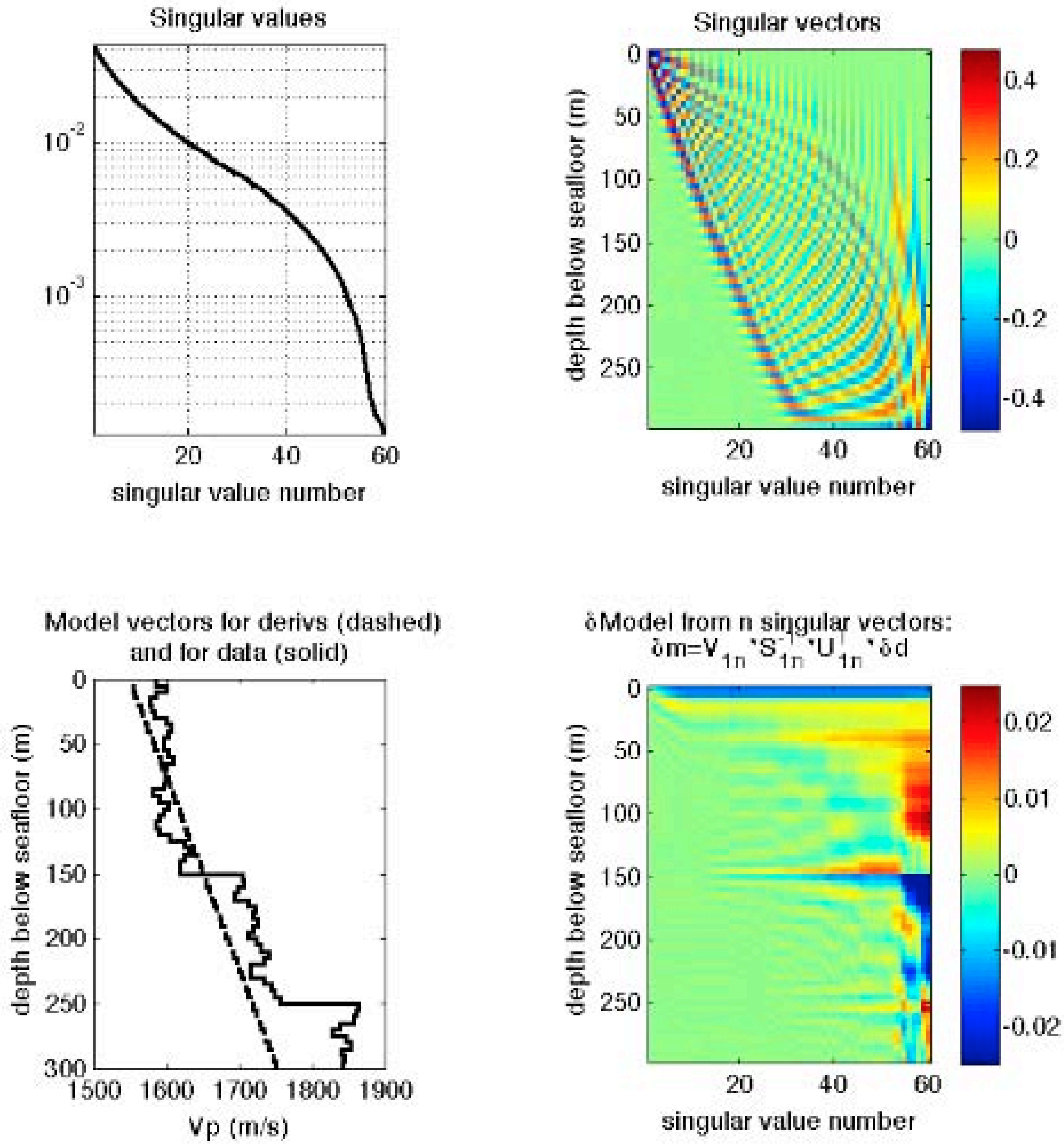

Figure 1: ***Upper left are the singular values, which cover only a few orders of magnitude. Upper right are the singular vectors. Lower left is the actual velocity model, showing smoother structure between two well defined discontinuities. The lower right shows the model perturbations as information from successive singular vectors is added. Notice the location of the first discontimuity at 150m and a hint of the second one at 250m.***

This "measured" data is the only place in the problem where this more complicated profile with discontinuities incorporated is used. The problem is locally linearized at the initial estimate to solve for a perturbation to update the model profile. The SVD analysis is based on the relationship between model perturbations and data residuals as the problem attempts to step from the smooth model profile

to the complicated one. The Jacobian matrix from the local linearization is decomposed into singular values and vectors to produce the results in this figure.

At upper left in Figure 1, note the singular values cover only a few orders of magnitude. However, the problem still requires regularization to avoid fitting structure in the data that is just noise. The set of singular vectors is in the upper right plot. The singular vectors corresponding to the largest singular values have the most structure right near the seafloor surface, and with more singular vectors then deeper structure is resolved. The singular vectors corresponding to the smaller singular values lose their meaning (and their structure in this plot), for the derivatives matrix was computed by a finite difference method that was limited in its accuracy. The plot at lower right in Figure 1 shows the solution for the model perturbation at different levels of regularization, by cumulatively including each additional singular vector. A given column of this plot is a choice of model perturbation to step from the smooth bottom profile in the lower left plot to the rough one that includes the discontinuities. With just the first few singular vectors included, one sees the strong response at the surface, just as in the singular vector plot above; similarly, more structure is resolved at depth with more singular values included. The 150m discontinuity is especially well resolved, and hints of the 250m one can be seen with the higher-numbered singular vectors. The information about the location of the discontinuities comes only from the "measured" data. The reflections seen in the data time-series at particular arrival times are conveniently mapped by means of the derivatives into perturbations at their corresponding depths. Lastly, as in the singular vectors plot, there is a region corresponding to the smallest singular values at which we run into the limits of the accuracy of the derivatives matrix which was computed by finite differences.

IMPACT/APPLICATIONS

This SVD based analysis is useful to gain understanding about the problem and to identify locations of the discontinuities, but ultimately the SVD approach did not fare well as the regularization type for the iterated inverse problem. Instead, minimizing a norm not of the model size (as in the SVD approach) but of the model smoothness, one solves for the model profile with the fewest features that matches the measured data to within the noise. The discontinuity locations must be specified to this problem - as taken, for example, from the SVD based analysis which mapped them from the reflection arrival times. But with this approach one can begin to distinguish when there is enough information in the data to resolve smooth variations in the model between discontinuities, for if there is not, one obtains straight lines between the discontinuities, either isovelocity or constant gradient. In the worse case, one obtains a solution with just a few homogeneous layers, just as one might have formulated the problem in a simpler way. But in better cases, this approach allows one to resolve smooth variations in the model between the stronger discontinuities, gaining more information from the same data.

RELATED PROJECTS

Our research is directly related to other programs studying effects of uncertainty in the environment, measurements, and models on acoustic propagation, and target detection and characterization.

REFERENCES

Menke, W., *Geophysical data analysis: discrete inverse theory*, Rev. ed., 285 p., Academic Press, San Diego, California, 1989.

Potty, G., J.H. Miller, J.F. Lynch, K.B. Smith, "Tomographic inversion for sediment parameters in shallow water," *J. Acoust. Soc. Am.*, **108**, pp973-986, 2000.

PUBLICATIONS

Odom, R.I. and A.A. Ganse, “The second order resolution operator of a nonlinear ocean acoustics inverse problem,”(abstract), *J. Acoust. Soc. Am.*, **125**, Pt.2 p2501, 2009.

Odom, R.I. and K. James, “Modeling gradients in ocean acoustics,” (abstract), *J. Acoust. Soc. Am.*, **125**, Pt.2 p2501, 2009.

Odom, R.I. and L. Blair, "Traveling Wave Modal Attenuation in Layered Media," *Eos Trans. AGU*, **90**(52), Fall Meet. Suppl., Abstract S52A-03.

To Be Submitted

Soukup, D.J., R.I. Odom, and J. Park, "A Modal Investigation odf Elastic Anisotropy in Shallow Water Environments: a study of anisotropy beyond VTI," *J. Acoust. Soc Am.*, to be submitted, 2010.

Soukup, D.J., R.I. Odom, and J. Park, “Coupled Modes, Range Dependence, and Sediment Anisotropy in Shallow Water Acoustic Propagation,” *J. Acoust. Soc Am.*, to be submitted, 2010.